Journal of Statistical Physics, Vol. 124, No. 5, September 2006 (© 2006)
DOI: 10.1007/s10955-006-9080-4

Exact Renormalization Group for the Brazovskii Model of Striped Patterns

Y. Shiwa[1]

Received August 12, 2005; accepted February 28, 2006
Published Online: September 1, 2006

We consider the coarse graining of the generalized Brazovskii free energy functional for striped patterns. The technique developed by Shankar for the Fermi liquids is combined with the irreducible version of the exact renormalization group to calculate the recursion relations for interaction vertices. We perform the one-loop calculations from this method taking the eight-point vertex into account.

KEY WORDS: Exact renormalization group, Brazovskii class, striped pattern, eight-point vertex function.

1. INTRODUCTION

The systems that undergo transitions from an isotropic, disordered phase to a nonuniform, spatially periodic phase are commonly referred to as Brazovskii systems (class). Examples include weakly anisotropic antiferromagnets,[(1)] liquid crystals near the isotropic-cholesteric[(2)] or the nematic-smectic C[(3)] transitions, pion condensates in neutron stars,[(4)] fluids near Rayleigh-Bénard convective instability,[(5)] and diblock copolymers.[(6)] An essential feature of these systems is that the ordered phase is described by the spatial period $2\pi/k_0$ and the fluctuation spectrum has a minimum at nonzero wave vectors $\boldsymbol{k}$ with $|\boldsymbol{k}| = k_0$, represented by a hypersphere in reciprocal space. Because of the large phase space for one-dimensional fluctuations in the direction transverse to the hypersphere, the Brazovskii systems are quite distinctive in comparison with the usual systems, where the periodic structure is determined by isolated points in reciprocal space and consequently the phase volume of fluctuations is small.

[1] Statistical Mechanics Laboratory, Kyoto Institute of Technology, Matsugasaki, Sakyo-ku, Kyoto 606-8585, Japan; e-mail: shiway@kit.ac.jp

In order to deal with the large fluctuations in the vicinity of a shell of nonzero wave vectors, the renormalization-group (RG) theory has been worked out by Hohenberg and Swift.[7] They implemented the Wilson's momentum-shell RG[8] with the techniques developed by Shankar for Fermi liquids,[9] which have a similar phase space involving a Fermi surface. It turns out that all interaction parameters are relevant (hence there is no analog of the familiar ϵ expansions of critical phenomena theory), and that, within the one-loop approximation, the recursion relations cannot be integrated to obtain the bulk (thermodynamic) quantities in certain parameters region. Thus the often successful RG techniques simply fail for the Brazovskii class.

Confronted with the above problems, we present in this paper an exact RG equation for the generating functional (Γ) of the one-particle irreducible correlation functions. The exact RG method[10,11] employs the coarse graining procedure whereby the fluctuating degrees of freedom with wave vectors in the range $|k - k_0| > \tilde{\Lambda}$ are averaged over to obtain the effective Γ for modes with $|k - k_0| \leq \tilde{\Lambda}$. The exact flow equations can be derived as formal identities from the functional integral which defines this coarse graining, and they are cast in the form of functional differential equations. Hence they describe the scale dependence of the free energy functional from which the physical properties at a given length or momentum scale can be computed.

Unfortunately, hindered by the complexity of the functional differential equations thus obtained, the practical application of the exact RG method has not yet been successful. Nevertheless, the future investigation of the generic feature of the formulation of the exact RG equations which permits non-perturbative approximations should lead to a deep insight into the nature of perturbative renormalizability of the Brazovskii model. After all, one of the main interest of the exact RG method is its potential ability to tackle the physics in non-perturbative regime.

This paper is organized as follows. In Sec. 2 we briefly summarize some basic definitions and representations of generating functionals that we use in the formulation. In Sec. 3 we construct the exact RG flow equations for the Brazovskii model, using the irreducible version of the exact RG in the form derived by Kopietz and Busche.[12] In Sec. 4 we make contact with the perturbative approach and rederive the one-loop recursion relations obtained first in Ref. 7. A higher-order truncation of an infinite hierarchy of differential equations is given in Sec. 5, and then this paper concludes with summary and discussion in Sec. 6.

2. GENERATING FUNCTIONALS

We consider a scalar field theory with a bare action

$$S\{\phi\} = S_0\{\phi\} + S_{\text{int}}\{\phi\}, \tag{1}$$

where, in the Brazovskii model, the free part is given by

$$S_0\{\phi\} = \frac{1}{2}\int_{|k-k_0|<\tilde{\Lambda}_0} \frac{d\boldsymbol{k}}{(2\pi)^d}\phi_{\boldsymbol{k}} G_0^{-1}(\boldsymbol{k})\phi_{-\boldsymbol{k}}, \tag{2}$$

$\phi_{\boldsymbol{k}} = \int d\boldsymbol{r} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\phi(\boldsymbol{r})$ being the Fourier transform of an order parameter $\phi(\boldsymbol{r})$ in d dimensions, and

$$G_0^{-1}(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}) \equiv r_0 + \bar{\xi}_0^2(k-k_0)^2, \tag{3}$$

with $k \equiv |\boldsymbol{k}|$. Here k_0 is the wavenumber selected by the *interacting* system. We assume that initially the order-parameter fields $\phi_{\boldsymbol{k}}$ with large momenta $|k-k_0| > \tilde{\Lambda}_0$ have been integrated out. The interaction part is

$$S_{\text{int}}\{\phi\} = \frac{1}{2}\int_{|k-k_0|<\tilde{\Lambda}_0} \frac{d\boldsymbol{k}}{(2\pi)^d}[\Sigma_{\tilde{\Lambda}_0}(\boldsymbol{k}) - \Sigma_c(\boldsymbol{k})]\phi_{\boldsymbol{k}}\phi_{-\boldsymbol{k}} + \cdots, \tag{4}$$

where the ellipsis denotes higher-order interactions, and $S_{\text{int}}\{\phi\}$ is a local function of the fields that is invariant under $\phi \to -\phi$.

The term $\Sigma_{\tilde{\Lambda}_0}(\boldsymbol{k})$ is the contribution to the irreducible self-energy from the high-momentum field with $|k-k_0| > \tilde{\Lambda}_0$. The function $\Sigma_c(\boldsymbol{k})$ is the counter term

$$\Sigma_c(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}) - \varepsilon_b(\boldsymbol{k}), \tag{5}$$

which takes into account that in the free action S_0 we have added it to the energy dispersion in the absence of interactions, $\varepsilon_b(\boldsymbol{k}) = r_b + \bar{\xi}_b^2(k-k_b)^2$, so that the free propagator $G_0(\boldsymbol{k})$ is singular for $k = k_0$, i.e., on the true 'Fermi surface', (not for $k = k_b$, the bare Fermi surface of the non-interacting system) at $r_0 = 0$. Note the counter term is not known a priori, and in fact requires the solution of the whole problem (i.e., the so-called mode selection problem). For our present purpose, however, it suffices to assume that k_0 is already calculated by some algorithm[2]

The generating functional of the connected Green function can be defined by

$$e^{\mathcal{G}_c\{J\}} = \frac{\int \mathcal{D}\{\phi\}\, e^{-S\{\phi\}+(J,\phi)}}{\int \mathcal{D}\{\phi\}\, e^{-S_0\{\phi\}}}, \tag{6}$$

where we have used the notation

$$(J,\phi) \equiv \int d\boldsymbol{r} J(\boldsymbol{r})\phi(\boldsymbol{r}). \tag{7}$$

The connected n-point functions are then written as

$$G_c^{(n)}(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n) \equiv \langle\phi(\boldsymbol{r}_1)\cdots\phi(\boldsymbol{r}_n)\rangle_c = \left.\frac{\delta^n\mathcal{G}_c\{J\}}{\delta J(\boldsymbol{r}_1)\cdots\delta J(\boldsymbol{r}_n)}\right|_{J=0}. \tag{8}$$

[2] The exact RG method has recently been applied in Ref. 13 to the fermionic analogue of this problem, i.e., to the calculation of the true Fermi surface.

Note that $\mathcal{G}_c\{J\}$ can be formally represented as follows in terms of functional differential operators

$$
\begin{aligned}
e^{\mathcal{G}_e\{J\}} &= \frac{\int \mathcal{D}\{\phi\}\, e^{-S_{\rm int}\{\phi\}-S_0\{\phi\}+(J,\phi)}}{\int \mathcal{D}\{\phi\}\, e^{-S_0\{\phi\}}} \\
&= e^{-S_{\rm int}\{\frac{\delta}{\delta J}\}}\, e^{\frac{1}{2}(J,G_0 J)} \\
&= e^{\frac{1}{2}(J,G_0 J)}\Big[e^{\frac{1}{2}(\frac{\delta}{\delta\phi},G_0\frac{\delta}{\delta\phi})}\, e^{-S_{\rm int}\{\phi\}}\Big]_{\phi=G_0 J},
\end{aligned}
\tag{9}
$$

where

$$
(A, G_0 B) \equiv \iint d\boldsymbol{r}_1 d\boldsymbol{r}_2 A(\boldsymbol{r}_1) G_0(\boldsymbol{r}_1 - \boldsymbol{r}_2) B(\boldsymbol{r}_2). \tag{10}
$$

To obtain the generating functional of the one-particle irreducible Green functions, we perform a Legendre transformation:

$$
\mathcal{L}\{\varphi\} = (\varphi, J) - \mathcal{G}_c\{J\{\varphi\}\}, \tag{11}
$$

where $J = J\{\varphi\}$ is defined as a function of φ via

$$
\varphi(\boldsymbol{r}) = \langle \phi(\boldsymbol{r})\rangle_J \equiv \frac{\delta \mathcal{G}_c\{J\}}{\delta J(\boldsymbol{r})}. \tag{12}
$$

We then define the generating functional by

$$
\Gamma\{\varphi\} \equiv \mathcal{L}\{\varphi\} - \frac{1}{2}\big(\varphi, G_0^{-1}\varphi\big), \tag{13}
$$

which gives the irreducible n-point correlation function by

$$
G_{ir}^{(n)}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n) = \frac{\delta^n \Gamma\{\varphi\}}{\delta\varphi(\boldsymbol{r}_1)\cdots\delta\varphi(\boldsymbol{r}_n)}\bigg|_{\varphi=0}. \tag{14}
$$

3. EXACT FLOW EQUATIONS

3.1. Coarse Graining

To integrate out degrees of freedom with momenta in the shell $\tilde{\Lambda} \le |k - k_0| < \tilde{\Lambda}_0$, we introduce the cutoff-dependent free propagator $G_0^{\tilde{\Lambda}_0,\tilde{\Lambda}}$ with the matrix elements in momentum space given by

$$
\big[G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\big]_{\boldsymbol{k},\boldsymbol{k}'} = (2\pi)^d \delta(\boldsymbol{k}+\boldsymbol{k}') G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}). \tag{15}
$$

Hence

$$
G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) = \frac{\theta(\Omega_k - \tilde{\Lambda}) - \theta(\Omega_k - \tilde{\Lambda}_0)}{r_0 + \bar{\xi}_0^2 (k-k_0)^2}, \tag{16}
$$

where $\theta(x)$ is the Heaviside function, and $\Omega_k \equiv |k - k_0|$.

The elimination of degrees of freedom in the momentum shell $\tilde{\Lambda} \leq \Omega_k < \tilde{\Lambda}_0$ corresponds to contracting all terms generated by expanding $e^{-S_{\rm int}}$ in Eq. (9) with the propagator $G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}$. The exact flow equation for $\Gamma\{\varphi\}$, denoted now as $\Gamma_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\}$, is then obtained;

$$\partial_{\tilde{\Lambda}}\Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0} = -\frac{1}{2}\mathrm{Tr}\Big\{\partial_{\tilde{\Lambda}}\big(G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^{-1}\Big[\big(G^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^2\mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}\big(1+G^{\tilde{\Lambda},\tilde{\Lambda}_0}\mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^{-1} + \big(G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^2\hat{\Sigma}^{\tilde{\Lambda},\tilde{\Lambda}_0}\big(1+G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\hat{\Sigma}^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^{-1}\Big]\Big\}, \tag{17}$$

with the initial condition (at $\tilde{\Lambda} = \tilde{\Lambda}_0$)

$$\Gamma^{\tilde{\Lambda}_0,\tilde{\Lambda}_0}\{\varphi\} = S_{\rm int}\{\varphi\}. \tag{18}$$

Here $\hat{\Sigma}^{\tilde{\Lambda},\tilde{\Lambda}_0}$ is defined as the field independent part of the second functional derivative of $\Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\}$

$$\frac{\delta^2\Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\}}{\delta\varphi_{\boldsymbol{k}}\delta\varphi_{\boldsymbol{k}'}} = [\hat{\Sigma}^{\tilde{\Lambda},\tilde{\Lambda}_0} + \mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\}]_{\boldsymbol{k},\boldsymbol{k}'} \tag{19}$$

so that $\mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi=0\} = 0$, and

$$G^{\tilde{\Lambda},\tilde{\Lambda}_0} \equiv \Big[\big(G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}\big)^{-1} + \hat{\Sigma}^{\tilde{\Lambda},\tilde{\Lambda}_0}\Big]^{-1} \tag{20}$$

is the interacting propagator.

Let us rewrite the right-hand side of Eq. (19) as

$$(2\pi)^d\delta(\boldsymbol{k}+\boldsymbol{k}')\,[\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) - \Sigma_c(\boldsymbol{k})] + \mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}_{\boldsymbol{k},\boldsymbol{k}'}\{\varphi\},$$

where the term $\Sigma_c(\boldsymbol{k})$ is due to the subtraction in Eq. (4); $\Sigma_{\tilde{\Lambda}_0}(\boldsymbol{k}) = \Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})$. Using then the relations[14,15]

$$-\frac{\partial}{\partial\tilde{\Lambda}}G_0^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) = \frac{\delta(\Omega_k-\tilde{\Lambda})}{r_0+\xi_0^2\Omega_k^2},$$

$$-\frac{\partial}{\partial\tilde{\Lambda}}[G^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})]^{-1} = \frac{\delta(\Omega_k-\tilde{\Lambda})}{r_0+\xi_0^2\Omega_k^2+\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k})}$$

with

$$G^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) = \frac{\theta(\Omega_k-\tilde{\Lambda})-\theta(\Omega_k-\tilde{\Lambda}_0)}{r_0+\xi_0^2\Omega_k^2+\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k})}, \tag{21}$$

Eq. (17) is reduced to

$$\begin{aligned}\partial_{\tilde{\Lambda}}\Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0} = &-\frac{1}{2}\int_{\boldsymbol{k}}\frac{\delta(\Omega_k-\tilde{\Lambda})}{r_0+\xi_0^2\tilde{\Lambda}^2+\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k})}\\ &\times\left[\mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0}(1+G^{\tilde{\Lambda},\tilde{\Lambda}_0}\mathcal{U}^{\tilde{\Lambda},\tilde{\Lambda}_0})^{-1}\right]_{\boldsymbol{k},-\boldsymbol{k}}\\ &-\frac{V}{2}\int_{\boldsymbol{k}}\delta(\Omega_k-\tilde{\Lambda})\ln\left[\frac{r_0+\xi_0^2\tilde{\Lambda}^2+\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k})}{r_0+\xi_0^2\tilde{\Lambda}^2}\right], \quad (22)\end{aligned}$$

where $\int_{\boldsymbol{k}} \equiv (2\pi)^{-d}\int d\boldsymbol{k}$ and V is the volume of the system.

3.2. Irreducible Vertices

We expand the generating functional $\Gamma_{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\} \equiv \Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\}$ in powers of φ,

$$\begin{aligned}\Gamma_{\tilde{\Lambda},\tilde{\Lambda}_0}\{\varphi\} = \Gamma^{(0)}_{\tilde{\Lambda},\tilde{\Lambda}_0} + &\sum_{n=1}^{\infty}\frac{1}{n!}\int_{\boldsymbol{k}_1}\cdots\int_{\boldsymbol{k}_n}(2\pi)^d\delta(\boldsymbol{k}_1+\cdots+\boldsymbol{k}_n)\\ &\times\Gamma^{(n)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\varphi_{\boldsymbol{k}_1}\cdots\varphi_{\boldsymbol{k}_n}, \quad (23)\end{aligned}$$

which defines the irreducible n-point vertices $\Gamma^{(n)}$. Note that only the even-point vertices are non-zero. Identifying the terms with the same powers of φ's on both sides of Eq. (22), we obtain an infinite hierarchy of flow equations for the irreducible vertices $\Gamma^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$. Notice that by construction the subtracted self-energy $\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k})$ is the irreducible two-point vertex

$$\Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) \equiv \Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k},-\boldsymbol{k}) = \Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k}).$$

It proves to be useful hereafter to redefine the two-point vertex up to a constant

$$\Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}) = r_0+\Sigma^{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})-\Sigma_c(\boldsymbol{k}). \quad (24)$$

We now explicitly give the exact flow equations for the first four nonzero vertices.

1. *Free energy*

   The vertex function $\Gamma^{(0)}$ describes the correction to the free energy from the interactions of the fields.

$$\partial_{\tilde{\Lambda}}\Gamma^{(0)}_{\tilde{\Lambda},\tilde{\Lambda}_0} = -\frac{V}{2}\int_{\boldsymbol{k}}\delta(\Omega_k-\tilde{\Lambda})\ln\left[\frac{\xi_0^2\tilde{\Lambda}^2+\Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})}{r_0+\xi_0^2\tilde{\Lambda}^2}\right]. \quad (25)$$

2. *Two-point vertex*

$$\partial_{\tilde{\Lambda}}\Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}_1) = -\frac{1}{2}\int_{\boldsymbol{k}}\frac{\delta(\Omega_k-\tilde{\Lambda})}{\xi_0^2\tilde{\Lambda}^2+\Gamma^{(2)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k})}\Gamma^{(4)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}_1,-\boldsymbol{k}_1,\boldsymbol{k},-\boldsymbol{k}). \quad (26)$$

3. *Four-point vertex*

$$\partial_{\tilde\Lambda}\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}_1,\cdot,\cdot,\boldsymbol{k}_4) = -\frac{1}{2}\int_{\boldsymbol{k}} \frac{\delta(\Omega_k-\tilde\Lambda)}{\bar\xi_0^2\tilde\Lambda^2+\Gamma^{(2)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k})}\Gamma^{(6)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}_1,\cdot,\cdot,\boldsymbol{k}_4,\boldsymbol{k},-\boldsymbol{k})$$

$$+\int_{\boldsymbol{k}} \frac{\delta(\Omega_k-\tilde\Lambda)}{\bar\xi_0^2\tilde\Lambda^2+\Gamma^{(2)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k})}\big[\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_2)$$

$$\times G^{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}+\boldsymbol{k}_1+\boldsymbol{k}_2)\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}+\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k},\boldsymbol{k}_3,\boldsymbol{k}_4)$$

$$+(\boldsymbol{k}_2\longleftrightarrow\boldsymbol{k}_3)+(\boldsymbol{k}_2\longleftrightarrow\boldsymbol{k}_4)\big]. \tag{27}$$

In Eq. (27), the last two terms in the square brackets are the same as the first term but with the indices swopped as indicated.

4. *Six-point vertex*

$$\partial_{\tilde\Lambda}\Gamma^{(6)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_6) =$$

$$-\frac{1}{2}\int_{\boldsymbol{k}} \frac{\delta(\Omega_k-\tilde\Lambda)}{\bar\xi_0^2\tilde\Lambda^2+\Gamma^{(2)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k})}\Gamma^{(8)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_6,\boldsymbol{k},-\boldsymbol{k})$$

$$+\int_{\boldsymbol{k}} \frac{\delta(\Omega_k-\tilde\Lambda)}{\bar\xi_0^2\tilde\Lambda^2+\Gamma^{(2)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k})}\sum_{\{I_1,I_2\}}^{15\text{ terms}}\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(I_1,-\boldsymbol{k}-\boldsymbol{K}_1,\boldsymbol{k})$$

$$\times G^{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}+\boldsymbol{K}_1)\Gamma^{(6)}_{\tilde\Lambda,\tilde\Lambda_0}(I_2,-\boldsymbol{k},\boldsymbol{k}+\boldsymbol{K}_1)$$

$$-\int_{\boldsymbol{k}} \frac{\delta(\Omega_k-\tilde\Lambda)}{\bar\xi_0^2\tilde\Lambda^2+\Gamma^{(2)}_{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k})}\sum_{\{I_1,I_2\},I_3}^{45\text{ terms}}\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(I_1,\boldsymbol{k},-\boldsymbol{k}-\boldsymbol{K}_1)$$

$$\times G^{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}+\boldsymbol{K}_1)\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(I_2,\boldsymbol{k}-\boldsymbol{K}_2,-\boldsymbol{k})$$

$$\times G^{\tilde\Lambda,\tilde\Lambda_0}(\boldsymbol{k}-\boldsymbol{K}_2)\Gamma^{(4)}_{\tilde\Lambda,\tilde\Lambda_0}(I_3,\boldsymbol{k}+\boldsymbol{K}_1,-\boldsymbol{k}+\boldsymbol{K}_2). \tag{28}$$

with $\boldsymbol{K}_j = \Sigma_{k\in I_j}\boldsymbol{k}_i$. Here the notation due to Morris[14] is used. Namely, $I_j (j = 1, 2, 3)$ are disjoint subsets ($I_i \cap I_j = \emptyset$ for $i \neq j$) of the momenta such that $\bigcup_{j=1}^{m} I_j = \{\boldsymbol{k}_1, \ldots, \boldsymbol{k}_6\}$ with $m = 2$ or 3. The sum over $\{I_1, I_2\}$ means a sum over all such disjoint subsets but pairs are to be counted only once. Graphical representation of Eq. (28) is shown in Fig. 1. The 15 terms of the first sum correspond to all the possibilities of choosing distinct subsets of four momenta out of the six available momenta $(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_6)$. The 45 terms in the second sum correspond to one half of the possibilities of picking subsets of four momenta out of the six momenta, and then picking subsets of two out of these four chosen momenta.

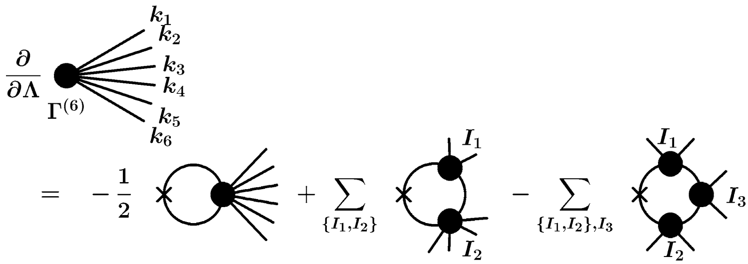

Fig. 1. Graphical illustration of the flow Eq. (28). The n-point irreducible vertex $\Gamma^{(n)}$ is drawn as a filled circle while the internal lines are full propagators G. The cross represents the restriction to momentum $\boldsymbol{k}$ such that $\Omega_k = \tilde{\Lambda}$.

3.3. Rescaling

After the elimination of degrees of freedom, we must put the coarse-grained generating functional $\Gamma^{\tilde{\Lambda},\tilde{\Lambda}_0}$ into the same form as the original Γ to complete the RG transformation. This can be achieved by rescaling the momenta and fields. We introduce dimensionless variables as follows:

$$k - k_0 = \tilde{\Lambda} q, \quad \tilde{\Lambda} = \tilde{\Lambda}_0 e^{-s},$$
$$\varphi_{\boldsymbol{k}} = \tilde{\Lambda}^{d_\varphi - d} Z_s^{1/2} \bar{\varphi}_{\boldsymbol{Q}} = \tilde{\Lambda}^{-3/2} Z_s^{1/2} \bar{\varphi}_{\boldsymbol{Q}}. \tag{29}$$

We have labeled the momentum $\boldsymbol{k}$ by its direction $\hat{\boldsymbol{n}} = \boldsymbol{k}/k$ and by the dimensionless variable $q \equiv (k - k_0)/\tilde{\Lambda}$, so that we now use the notation $\boldsymbol{Q} \equiv (\hat{\boldsymbol{n}}, q)$ instead of $\boldsymbol{k}$. Using $\boldsymbol{Q}$ istead of $\boldsymbol{k}$ as integration variables, we have[3]

$$\int_{\boldsymbol{k}} = 2\nu_0 \tilde{\Lambda} \int_{\boldsymbol{Q}} \equiv 2\nu_0 \tilde{\Lambda} \int \frac{dS_{\boldsymbol{n}}}{S_d} \int dq \tag{30}$$

where $\nu_0 = S_d k_0^{d-1}/(2\pi)^d$, and $dS_{\boldsymbol{n}}$ is the surface element, S_d being the surface area of the unit sphere, $S_d = 2\pi^{d/2}/\Gamma(d/2)(\Gamma(z)$ is the gamma function). Note also that the scaling dimension d_φ of the field $\varphi(\boldsymbol{r})$ in real space is given by $d_\varphi = d - 3/2$ in contrast to the case of the φ^4-theory for which $d_\varphi = (d-2)/2$ (see Appendix). The scale-dependent factor Z_s is a multiplicative 'wave function' (φ) renormalization factor.

The above rescaling of momenta and fields (29) implies the renormalized vertices, which should now be considered as functions of the scaling variables s

[3] In the Jacobian, $J(\hat{\boldsymbol{n}}, q)$, associated with the transformation $\boldsymbol{k} = (\hat{\boldsymbol{n}}, k) \to \boldsymbol{Q} = (\hat{\boldsymbol{n}}, q)$, we have neglected the factor $\tilde{\Lambda}/\boldsymbol{k}_0$ in comparison to q^{-1} (noting that $\tilde{\Lambda}_0 \approx k_0$);

$$J(\hat{\boldsymbol{n}}, q) = (k_0 + \tilde{\Lambda} q)^{d-1} \tilde{\Lambda} \approx k_0^{d-1} \tilde{\Lambda}.$$

and $\boldsymbol{Q}_j$ and hence are denoted as $\Gamma_s^{(2n)}(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_{2n})$, become

$$\Gamma_s^{(2n)}(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_{2n}) = \tilde{\Lambda}^{2n-1}\left(\frac{Z_s}{\tilde{\Lambda}^3}\right)^n \Gamma^{(2n)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_{2n}), \tag{31}$$

where the factor $2n - 1$ comes from the factor $\int_{\boldsymbol{k}_1} \cdots \int_{\boldsymbol{k}_{2n}} \delta(\boldsymbol{k}_1 + \cdots \boldsymbol{k}_{2n})$ in (23). Consequently, the flow equations of the renormalized vertices $\Gamma_s^{(2n)}$ follow in the straightforward way. We obtain

$$\begin{aligned}\partial_s \Gamma_s^{(2n)}(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_{2n}) &= (n + 1 - n\eta_s - \boldsymbol{Q}_i \cdot \nabla_{\boldsymbol{Q}_i})\Gamma_s^{(2n)}(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_{2n}) \\ &\quad - (Z_s/\tilde{\Lambda})^n \partial_{\tilde{\Lambda}} \Gamma^{(2n)}_{\tilde{\Lambda},\tilde{\Lambda}_0}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_{2n}).\end{aligned} \tag{32}$$

Here η_s is the 'anomalous' dimension given by

$$\eta_s = -\partial_s \ln Z_s,$$

and the second term on the right-hand side is a contribution from the mode-elimination transformation, which is given in the previous subsection.

4. RG AT ONE LOOP

4.1. Relevant Couplings

Due to the argument by Hohenberg and Swift,[7] which borrows from the one given by Shankar[9] for the Fermi liquid, we know that only a part of the n-point vertex functions ($n > 2$) whose wave vectors are equal and opposite in pairs with their magnitude equal to k_0 survives the iterated application of RG transformations. (For an overview on the essential ideas of the technique advanced by Shankar as applied to Brazovskii class, see Appendix, where a caveat on this point is given as well.) We call such a part the relevant vertex function. We thus separate the vertex function $\Gamma_s^{(m)}(m > 2)$ into a relevant and an irrelevant part:

$$\begin{aligned}&\Gamma_s^{(m)}((\hat{\boldsymbol{n}}_1, q_1), (\hat{\boldsymbol{n}}_2, q_2), \ldots, (\hat{\boldsymbol{n}}_{m-1}, q_{m-1}), (\hat{\boldsymbol{n}}_m, q_m)) \\ &\quad = \Gamma_s^{(m)}((\hat{\boldsymbol{n}}_1, 0), (-\hat{\boldsymbol{n}}_1, 0), \ldots, (\hat{\boldsymbol{n}}_m, 0), (-\hat{\boldsymbol{n}}_m, 0)) \\ &\qquad + \tilde{\Gamma}_s^{(m)}((\hat{\boldsymbol{n}}_1, q_1), (\hat{\boldsymbol{n}}_2, q_2), \cdots, (\hat{\boldsymbol{n}}_{m-1}, q_{m-1}), (\hat{\boldsymbol{n}}_m, q_m)).\end{aligned} \tag{33}$$

For simplicity we hereafter use the notations

$$\begin{aligned}u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2) &= \Gamma_s^{(4)}((\hat{\boldsymbol{n}}_1, 0), (-\hat{\boldsymbol{n}}_1, 0), (\hat{\boldsymbol{n}}_2, 0), (-\hat{\boldsymbol{n}}_2, 0)), \\ w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3) &= \Gamma_s^{(6)}((\hat{\boldsymbol{n}}_1, 0), (-\hat{\boldsymbol{n}}_1, 0), (\hat{\boldsymbol{n}}_2, 0), (-\hat{\boldsymbol{n}}_2, 0), (\hat{\boldsymbol{n}}_3, 0), (-\hat{\boldsymbol{n}}_3, 0)), \\ t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}_4) &= \Gamma_s^{(8)}((\hat{\boldsymbol{n}}_1, 0), (-\hat{\boldsymbol{n}}_1, 0), (\hat{\boldsymbol{n}}_2, 0), (-\hat{\boldsymbol{n}}_2, 0), \\ &\quad \times (\hat{\boldsymbol{n}}_3, 0), (-\hat{\boldsymbol{n}}_3, 0), (\hat{\boldsymbol{n}}_4, 0), (-\hat{\boldsymbol{n}}_4, 0)),\end{aligned} \tag{34}$$

etc. Because the RG flow rapidly approaches the manifold spanned by the relevant couplings, for a one-loop approximation presented below, the parts $\tilde{\Gamma}^{(m)}$ can be ignored.

4.2. One-Loop Calculations

To one-loop order the flow equations for the vertex functions are then given in terms of the coupling functions (34). We find, for the function $r_s(\hat{\boldsymbol{n}}) \equiv \Gamma_s^{(2)}((\hat{\boldsymbol{n}}, 0), (-\hat{\boldsymbol{n}}, 0))$,

$$(\partial_s - 2 + \eta_s) r_s(\hat{\boldsymbol{n}}_1) = \nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}), \tag{35}$$

where

$$g(\boldsymbol{Q}) \equiv \frac{\delta(q-1)}{r_s(\hat{\boldsymbol{n}}) + Z_s \bar{\xi}_0^2},$$

and Eq. (30) is implied for the integral.

For $u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2)$, it turns out that one must distinguish between the case $\hat{\boldsymbol{n}}_1 \neq \hat{\boldsymbol{n}}_2$ and the case $\hat{\boldsymbol{n}}_1 = \hat{\boldsymbol{n}}_2$. Then, for $\hat{\boldsymbol{n}}_1 \neq \hat{\boldsymbol{n}}_2$,

$$\begin{aligned}(\partial_s - 3 + 2\eta_s) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2) = {}& \nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}) \\ & - 2\nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) u_s(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_1) u_s(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_2) G_s(\mathbf{Q}),\end{aligned} \tag{36}$$

and

$$\begin{aligned}(\partial_s - 3 + 2\eta_s) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1) = {}& \nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}) \\ & - 4\nu_0 \int_{\mathbf{Q}} g(\mathbf{Q}) [u_s(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}_1)]^2 G_s(\boldsymbol{Q}),\end{aligned} \tag{37}$$

where

$$G_s(\mathbf{Q}) \equiv \frac{\theta(q-1) - \theta(q - e^s)}{r_s(\hat{\boldsymbol{n}}) + Z_s \bar{\xi}_0^2 q^2}.$$

Similarly, for w_s we find (with $\hat{\boldsymbol{n}}_i \neq \hat{\boldsymbol{n}}_j$ for $i \neq j$)

$$\begin{aligned}(\partial_s - 4 + 3\eta_s) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3) = {}& \nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) \\ & - 6\nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) w_s(\hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) G_s(\boldsymbol{Q})\end{aligned}$$

$$+ 6\nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) u_s(\hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}) u_s(\hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) [G_s(\boldsymbol{Q})]^2, \tag{38}$$

$$\begin{aligned}(\partial_s - 4 + 3\eta_s) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_3) &= \nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) \\ &- 8\nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) G_s(\mathbf{Q}) \\ &- 2\nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) u_s(\hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) G_s(\mathbf{Q}) \\ &+ 12\nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) [u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}})]^2 u_s(\hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}) [G_s(\mathbf{Q})]^2,\end{aligned} \tag{39}$$

$$\begin{aligned}(\partial_s - 4 + 3\eta_s) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1) &= \nu_0 \int_{\mathbf{Q}} g(\mathbf{Q}) t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) \\ &- 18\nu_0 \int_{\boldsymbol{Q}} g(\mathbf{Q}) u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}) G_s(\boldsymbol{Q}) \\ &+ 36\nu_0 \int_{\boldsymbol{Q}} g(\boldsymbol{Q}) [u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}})]^3 [G_s(\boldsymbol{Q})]^2.\end{aligned} \tag{40}$$

4.3. Preaverage Approximation and Recursion Relations

In order to proceed further we resort to truncation of the hierarchical set of flow equations given in the previous subsection. Let us first set $\Gamma^{(n)} = 0$ for $n \geq 8$, expecting this is sufficient to describe the first-order transition that will occur in our model. Furthermore, we assume that the angle dependence of u_s and w_s is negligible and hence they are practically constant.[7] This is equivalent to replacing the coupling functions by the angle-averaged quantities in the flow equations (called preaverage approximation). We thus introduce the notations

$$\begin{aligned}\tilde{r} &= \bar{\xi}_0^{-2} r_s(\hat{\boldsymbol{n}}), \\ \tilde{u}_a &= \nu_0 \bar{\xi}_0^{-4} u_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2), \quad \tilde{u}_b = \nu_0 \bar{\xi}_0^{-4} u_s(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}), \\ \tilde{w}_a &= \nu_0^2 \bar{\xi}_0^{-6} w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3), \quad \tilde{w}_b = \nu_0^2 \bar{\xi}_0^{-6} w_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2), \\ \tilde{w}_c &= \nu_0^2 \bar{\xi}_0^{-6} w_s(\hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}, \hat{\boldsymbol{n}}),\end{aligned} \tag{41}$$

with $\hat{\boldsymbol{n}}_i \neq \hat{\boldsymbol{n}}_j$ for $i \neq j$, where we have inserted the cosmetic factors $\bar{\xi}_0^{-2}$, etc., simply so as to make the resulting recursion equations terse. Moreover, the wave function renormalization is assumed to be unnecessary (hence $Z_s = 1$), which is somewhat ad hoc at this point but turns out to be adequate for the one-loop approximations that we carry out subsequently.

Upon carrying out the integrals with the help of the identities[14]

$$\int dq\delta(q-1)\theta(q-1) = \frac{1}{2},$$

$$\int dq\delta(q-1)[\theta(q-1)]^2 = \frac{1}{3},$$

we can now write down the recursion relations for the quantities defined by Eq. (41). They read

$$\begin{aligned}
(\partial_s - 2)\tilde{r} &= \frac{\tilde{u}_a}{\tilde{r}+1},\\
(\partial_s - 3)\tilde{u}_a &= \frac{\tilde{w}_a}{\tilde{r}+1} - \frac{\tilde{u}_a^2}{(\tilde{r}+1)^2},\\
(\partial_s - 3)\tilde{u}_b &= \frac{\tilde{w}_b}{\tilde{r}+1} - \frac{2\tilde{u}_a^2}{(\tilde{r}+1)^2},\\
(\partial_s - 4)\tilde{w}_a &= -\frac{3\tilde{u}_a\tilde{w}_a}{(\tilde{r}+1)^2} + \frac{2\tilde{u}_a^3}{(\tilde{r}+1)^3},\\
(\partial_s - 4)\tilde{w}_b &= -\frac{\tilde{u}_a\tilde{w}_b}{(\tilde{r}+1)^2} - \frac{4\tilde{u}_a\tilde{w}_a}{(\tilde{r}+1)^2} + \frac{4\tilde{u}_a^3}{(\tilde{r}+1)^3},\\
(\partial_s - 4)\tilde{w}_c &= -\frac{9\tilde{u}_a\tilde{w}_b}{(\tilde{r}+1)^2} + \frac{12\tilde{u}_a^3}{(\tilde{r}+1)^3}.
\end{aligned} \tag{42}$$

As is evident from the above recursion relations, vertex quantities are all relevant variables with respect to $\tilde{r} = \tilde{u}_j = \tilde{w}_j = 0$.[7] As a consequence, suffering from the absence of a small parameter, we have no controlled ϵ expansion, in distinction to a successful application of RG methods to critical phenomena. An important observation[7,16] is in order here. After the RG transformation, the Γ still has the form (23) when expressed in terms of the $\tilde{r}(s)$, $\tilde{u}_j(s)$, $\tilde{w}_j(s)$ and the corresponding field, $\tilde{\varphi}(s)$, with the upper cut-off $\tilde{\Lambda}_0$ (since the cut-off is restored to its original value by the rescaling transformation). Let us now make a change of variables

$$r(s) = e^{-2s}\tilde{r}(s), \quad u_j(s) = e^{-3s}\tilde{u}_j(s), \quad w_j(s) = e^{-4s}\tilde{w}_j(s) \tag{43}$$

supplemented with

$$\varphi(s) = e^{3s/2}\tilde{\varphi}(s). \tag{44}$$

Then the Γ expressed in terms of these new variables takes precisely the same form as (23) with now the upper cut-off $\tilde{\Lambda} = \tilde{\Lambda}_0\, e^{-s}$. Therefore, this Γ can be regarded as a coarse-grained Γ obtained by eliminating the modes in the interval $\tilde{\Lambda} < |k - k_0| < \tilde{\Lambda}_0$.

If we define $\Lambda = e^{-s}$, these r, u_j and w_j satisfy the recursion relations which are

$$\begin{aligned}
\frac{\partial r}{\partial \Lambda} &= -\frac{u_a}{r + \Lambda^2}, \\
\frac{\partial u_a}{\partial \Lambda} &= -\frac{w_a}{r + \Lambda^2} + \frac{u_a^2}{(r + \Lambda^2)^2}, \\
\frac{\partial u_b}{\partial \Lambda} &= -\frac{w_b}{r + \Lambda^2} + \frac{2u_a^2}{(r + \Lambda^2)^2}, \\
\frac{\partial w_a}{\partial \Lambda} &= \frac{3u_a w_a}{(r + \Lambda^2)^2} - \frac{2u_a^3}{(r + \Lambda^2)^3}, \\
\frac{\partial w_b}{\partial \Lambda} &= \frac{4u_a w_a}{(r + \Lambda^2)^2} + \frac{u_a w_b}{(r + \Lambda^2)^2} - \frac{4u_a^3}{(r + \Lambda^2)^3}, \\
\frac{\partial w_c}{\partial \Lambda} &= \frac{9u_a w_b}{(r + \Lambda^2)^2} - \frac{12u_a^3}{(r + \Lambda^2)^3}.
\end{aligned} \tag{45}$$

These recursion relations are exactly in the same form as those obtained by Hohenberg and Swift[(7)] (we have corrected the error in Eq. (A24f) of Ref. 7, where the factor 9 in the right-hand side of the last equation of Eq. (45) is replaced with the factor 8). Note, however, that $\Lambda = \tilde{\Lambda}/\tilde{\Lambda}_0$ in our case, whereas in Ref. 7 their $\Lambda =$ our $\tilde{\Lambda}$. The difference stems from our definition (31) of the dimensionless (with respect to momentum) vertices. In fact, with the change into the dimensional variables (which we call HS-variables)

$$\begin{aligned}
&r_{\mathrm{HS}} \equiv r\tilde{\Lambda}_0^2, \quad u_{\mathrm{HS}j} \equiv u_j\tilde{\Lambda}_0^3, \quad w_{\mathrm{HS}j} \equiv w_j\tilde{\Lambda}_0^4, \qquad (j = a, b, c), \\
&\Lambda_{\mathrm{HS}} \equiv \Lambda\tilde{\Lambda}_0 (= \tilde{\Lambda})
\end{aligned} \tag{46}$$

the recursion relations for the HS-variables take exactly the same mathematical structure as of Eq. (45) with the scale variable Λ_{HS} varying in the range $[0, \tilde{\Lambda}_0]$.

Next we present some numerical results of Eq. (45). For ease of comparison with the results of Ref. 7, we employ the HS-variables in the analysis. We solve the recursion relations (45) numerically with the initial conditions at $s = 0$, i.e., at $\Lambda_{\mathrm{HS}} = \tilde{\Lambda}_0 = \infty$ (where we have set $\tilde{\Lambda}_0 = \infty$) as follows:

$$r_{\mathrm{HS}} = \tau, \quad u_{\mathrm{HS}j} = 1, \quad w_{\mathrm{HS}j} = 0 \tag{47}$$

with $j = a, b, c$. However, we find that the denominator in the Eq. (45), $r_{\mathrm{HS}}(\Lambda_{\mathrm{HS}}, \tau) + \Lambda_{\mathrm{HS}}^2$, contains zero at some $\tau < 0$, hence the solutions are not well-defined for all τ and Λ_{HS}. The region in the parameter space $(\tau, \Lambda_{\mathrm{HS}})$ where

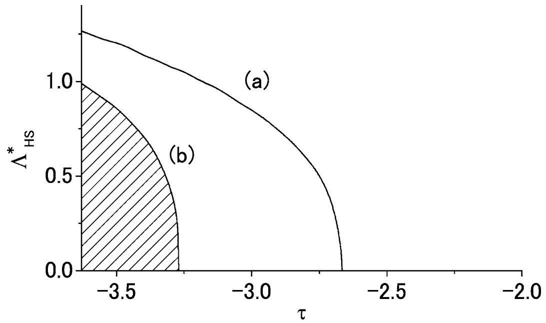

Fig. 2. (a) The function $\Lambda^*_{\rm HS}(\tau)$ at which $r_{\rm HS}(\Lambda_{\rm HS}, \tau) + \Lambda^2_{\rm HS}$ has zeros. In the underlying region the solutions of the recursion relations (45) are not defined. (b) Same as (a) but for the Brazovskii model with the eight vertex function. The recursion relations (50) have a singularity in the shaded region.

such singularities occur in the recursion relations is depicted in Fig. 2(a). Figure 3 (dotted lines) show how the 'bulk' vertices $r_{\rm HS}(\Lambda_{\rm HS} = 0)$, $u_{\rm HSb}(\Lambda_{\rm HS} = 0)$ and $w_{\rm HSc}(\Lambda_{\rm HS} = 0)$ behave as a function of τ when $r_{\rm HS} + \Lambda^2_{\rm HS}$ remains positive. Notice here that the coupling constant $r_{\rm HS}$ remains nonvanishing at the bulk limit

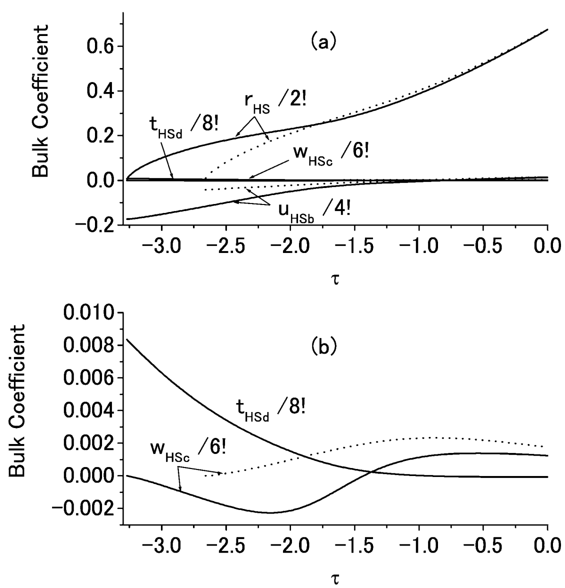

Fig. 3. Bulk vertex constants as a function of τ. They are obtained by solving the recursion relations with the initial conditions: $r_{\rm HS} = \tau, u_{{\rm HS}j} = 1, w_{{\rm HS}j} = 0, t_{{\rm HS}j} = 0$ where $j = a, b, c, d$; the dotted lines represent the solutions for (45), while the continuous curves for (50). On the scale in the panel (a), higher-order vertices are almost indistinguishable. For an enlargement, see the panel (b).

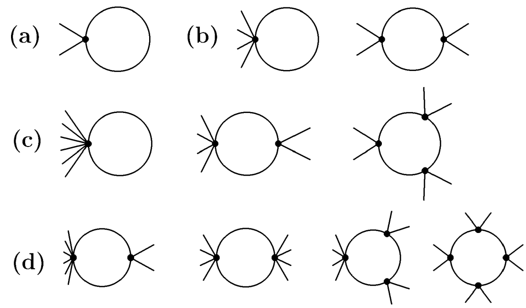

Fig. 4. One-loop diagrams contributing to irreducible vertex functions; diagrams contributing to $\Gamma^{(2)}$ (a), $\Gamma^{(4)}$ (b), $\Gamma^{(6)}$ (c), and to $\Gamma^{(8)}$ (d).

$\Lambda_{\text{HS}} = 0$. This means that the correlation length of the system is finite, signalling a (fluctuation-induced) first-order phase transition that belongs to the Brazovskii class.[1] It thus seems that the rescaling (43) and (44) is physically more plausible than the conventional scaling given in Eq. (29).

In passing we note that the behavior of coefficients of the bulk free energy that we obtain by solving the recursion relations (45) is in more agreement with the result of what Ref. 7 called the 'phenomenological' theory[4], owing to our corrected recursion relation for the coupling constant w_c.

5. HIGHER ORDER TRUNCATION

As we have shown in the preceding section, the solutions of Eq. (45) do not exist for all τ and Λ. It is therefore interesting to investigate whether inclusion of higher-order vertices not taken into account in the preceding calculation would be free from such difficulty. To that end we repeat the RG calculation by retaining the eight-point vertex function. The corresponding diagrams at one-loop order are shown in Fig. 4. Besides the notations (41), we define

$$\tilde{t}_a = \nu_0^3 \xi_0^{-8} t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3, \hat{\boldsymbol{n}}_4), \quad \tilde{t}_b = \nu_0^3 \xi_0^{-8} t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3),$$
$$\tilde{t}_c = \nu_0^3 \xi_0^{-8} t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2), \quad \tilde{t}_d = \nu_0^3 \xi_0^{-8} t_s(\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_1), \tag{48}$$

with $\hat{\boldsymbol{n}}_i \neq \hat{\boldsymbol{n}}_j$ for $i \neq j$, and make an additional change of variables

$$t_j = e^{-5s} \tilde{t}_j, \quad t_{\text{HS}j} = t_j \tilde{\Lambda}_0^5. \tag{49}$$

[4] Their phenomenological coarse-graining procedure consists of (i) averaging the free energy over the range $\tilde{\Lambda} < |k - k_0| < \tilde{\Lambda}_0$ using the Hartree diagrams, and (ii) no rescaling of momenta and field is performed.

We then obtain, within the same approximations as before,

$$
\begin{aligned}
\frac{\partial r}{\partial \Lambda} &= -\frac{u_a}{r+\Lambda^2}, \\
\frac{\partial u_a}{\partial \Lambda} &= -\frac{w_a}{r+\Lambda^2} + \frac{u_a^2}{(r+\Lambda^2)^2}, \\
\frac{\partial u_b}{\partial \Lambda} &= -\frac{w_b}{r+\Lambda^2} + \frac{2u_a^2}{(r+\Lambda^2)^2}, \\
\frac{\partial w_a}{\partial \Lambda} &= -\frac{t_a}{r+\Lambda^2} + \frac{3u_a w_a}{(r+\Lambda^2)^2} - \frac{2u_a^3}{(r+\Lambda^2)^3}, \\
\frac{\partial w_b}{\partial \Lambda} &= -\frac{t_b}{r+\Lambda^2} + \frac{4u_a w_a}{(r+\Lambda^2)^2} + \frac{u_a\, w_b}{(r+\Lambda^2)^2} - \frac{4u_a^3}{(r+\Lambda^2)^3}, \\
\frac{\partial w_c}{\partial \Lambda} &= -\frac{t_c}{r+\Lambda^2} + \frac{9u_a\, w_b}{(r+\Lambda^2)^2} - \frac{12u_a^3}{(r+\Lambda^2)^3}, \\
\frac{\partial t_a}{\partial \Lambda} &= \frac{4u_a\, t_a}{(r+\Lambda^2)^2} + \frac{3w_a^2}{(r+\Lambda^2)^2} - \frac{12u_a^2\, w_a}{(r+\Lambda^2)^3} + \frac{6u_a^4}{(r+\Lambda^2)^4}, \\
\frac{\partial t_b}{\partial \Lambda} &= \frac{4u_a\, t_a}{(r+\Lambda^2)^2} + \frac{2u_a\, t_b}{(r+\Lambda^2)^2} + \frac{4w_a^2}{(r+\Lambda^2)^2} + \frac{2w_a\, w_b}{(r+\Lambda^2)^2} \\
&\quad - \frac{20u_a^2\, w_a}{(r+\Lambda^2)^3} - \frac{2u_a^2\, w_b}{(r+\Lambda^2)^3} + \frac{12u_a^4}{(r+\Lambda^2)^4}, \\
\frac{\partial t_c}{\partial \Lambda} &= \frac{u_a\, t_c}{(r+\Lambda^2)^2} + \frac{9u_a\, t_b}{(r+\Lambda^2)^2} + \frac{18w_a\, w_b}{(r+\Lambda^2)^2} \\
&\quad - \frac{36u_a^2\, w_a}{(r+\Lambda^2)^3} - \frac{18u_a^2\, w_b}{(r+\Lambda^2)^3} + \frac{36u_a^4}{(r+\Lambda^2)^4}, \\
\frac{\partial t_d}{\partial \Lambda} &= \frac{16u_a\, t_c}{(r+\Lambda^2)^2} + \frac{36w_b^2}{(r+\Lambda^2)^2} - \frac{144u_a^2\, w_b}{(r+\Lambda^2)^3} + \frac{144u_a^4}{(r+\Lambda^2)^4}\,.
\end{aligned} \tag{50}
$$

Again as before, when Eq. (50) are reexpressed in terms of the HS-variables, they yield precisely the same form as Eq. (50). However, we find that the solutions of the recursion Eq. (50) are still not well-defined for all τ and Λ [see Figs. 2(b) and 3]. Nevertheless, we note that the unphysical parameter region is rendered shrunk by including the eight-vertex interactions.

6. SUMMARY AND DISCUSSION

Thus, even with φ^8-interactions taken into consideration, the RG recursion relations contain singularities for $\tau < 0$, and the difficulty to use RG method for Brazovskii model remains unresolved.

As we mentioned earlier, Eq. (50) exhibits the ‘bulk’ solutions only for limited values of parameters (τ, Λ). This is related to the presence of the pole of the renormalized propagator $[\propto (r + \Lambda^2)^{-1}]$. This pole is connected with the marginal coupling constant r, and including an irrelevant coupling which measures the leading deviation from the energy dispersion (3) is likely to remove it from the parameter space. It would hence be of interest to check this assumption by studying the effect of such (possibly dangerous) irrelevant couplings, which is left for future investigation.

We should emphasize that this same feature, i.e., the existence of the singularity (for $\Lambda = \Lambda^*$) at which the renormalized propagator diverges is also known in the RG approach to the coarse-grained free energy of the two-phase coexistence in the Ising model.[16] The line for $\Lambda^* \to 0$ gives the effective spinodal curve. However, this is thought to be incorrect for real systems,[16,17] and is an artifact of the approximate way of coarse-graining using the finite order ϵ-expansion. Thus the finite order RG method cannot correctly explain all aspects of first-order phase transitions even in the case of Ising universality class, for which one finds two relevant couplings for $\epsilon = 4 - d > 0$. [Recall the recursion relations for the ϕ^4-theory parametrized by quadratic and quartic coupling constants, $\tilde{r}$, and $\tilde{u}$, respectively. They read[18] in appropriate units

$$\partial_s \tilde{r} = 2\tilde{r} + A\frac{\tilde{u}}{\tilde{r}+1},$$

$$\partial_s \tilde{u} = \epsilon\tilde{u} - B\frac{\tilde{u}^2}{(\tilde{r}+1)^2},$$

where A and B are some positive constants. The above equations should be compared with Eq. (42).]

It might also be worth trying a systematic truncation of the flow equations in the number of vertex constants *a la* RG calculations by Diaz-Guilera[19] for the self-organized critical phenomena, for which one also encounters an infinite number of relevant coupling constants. Although being straightforward, it would be prohibitively laborious and we leave it for future discussion.

So far in our RG treatment of the Brazovskii model (1), we have had critical phenomena in mind. Another phenomenon of great interest is the process of pattern formation far from the critical point. When the system is suddenly quenched from a homogeneous high-temperature phase to an ordered phase where the initial state is thermodynamically unstable, the system develops a labyrinthine domain morphology consisting of locally ordered stripes. As time increases, stripes which are initially randomly oriented align in parallel, thereby creating an increasingly ordered pattern. In the late stages of this coarsening process, there is evidence[20] for the type of scaling behavior and universality usually encountered in critical phenomena. Namely, the structure factor, $S(k, t) \equiv \langle \varphi_{\boldsymbol{k}}(t)\varphi_{-\boldsymbol{k}}(t)\rangle$, which is a Fourier transform of the order-parameter correlation function, is found to have a scaling form

$$S(k, t) = \ell(t)\ f(|k - k_0|\ell(t)) \tag{51}$$

with the characteristic domain size $\ell(t)$, where $\ell(t)$ increases as a power of time

$$\ell(t) \sim t^n.$$

Under the rescaling of the RG transformations [cf. (A1) and (A2) below]:

$$q = q'/b, \quad t = b^z t' \quad \text{with } z = 1/n,$$
$$\varphi_q(t) = b^{\hat{\zeta}} \varphi'_{q'}(t'),$$

where we have introduced the shorthand φ_q for $\varphi_{\boldsymbol{k}}$, the scaling (51) of the structure factor requires

$$\hat{\zeta} = 1/2 \tag{52}$$

Hence the wave function renormalization factor Z_s [notice that $b = \tilde{\Lambda}_0/\tilde{\Lambda}$] takes the value $Z_s = b^{-2}$, from which it follows that the anomalous dimension is

$$\eta_s = -\frac{\partial \ln Z_s}{\partial \ln b} = 2\ . \tag{53}$$

This should be compared with the case of Ising model, for which a result $\eta s = 2 - d$ is obtained.[21] With the formula (53) all the calculations carried out in the previous sections (Secs. 4.3 and 5) can be simply repeated. Now we see that the coarse-grained Γ functional with the upper cut-off $\tilde{\Lambda}$ is expressed in terms of the new variables defined by

$$r(s) = \tilde{r}(s), \quad u_j(s) = e^s \tilde{u}_j, \quad w_j(s) = e^{2s} \tilde{w}_j(s),$$
$$t_j(s) = e^{3s} \tilde{t}_j(s), \quad \varphi(s) = e^{s/2} \tilde{\varphi}(s), \tag{54}$$

in place of the definitions (43), (44) and (49). Then, interestingly, the recursion relation takes precisely the form (45) or (50) when expressed in terms of the running variables (54).

It is important to note that the last result just given above is based upon the assumption of simple scaling (51). At present Eq. (51) is nothing more than a successful phenomenological formula to fit the data of growth kinetics of stripe patterns. There is no first-principle derivation of this dynamical scaling form, with few efforts in this direction,[22,23] and its validity still needs to be questioned.

We finally mention that many dynamical phenomena in Brazovskii systems (such as coarsening processes of stripe patterns and the motion of topological defects[(24)]) are very much slow. However at present no successful theory is yet available to deal with the problem in general. Apparently the success of the RG transformation for the free energy functional is a prerequisite to understanding the mechanism of the dynamical behavior of any system in the Brazovskii class by RG methods.

APPENDIX: MOMENTUM-SHELL RG

Wilson's renormalization-group method[(8)] involves two steps of transformation. They are

1. Mode elimination: integrating out the fast modes $\varphi_{\boldsymbol{k}}$ with momentum $\boldsymbol{k}$ satisfying $|k - k_0| > \Lambda_0/b$ to reduce the cutoff from $\tilde{\Lambda}_0$ to $\tilde{\Lambda}_0/b$ with $b > 1$.
2. Rescaling of momenta and field: writing $\boldsymbol{k} = (k_0 + q)\boldsymbol{n}$ where $\boldsymbol{n}$ is the unit vector, the momenta rescaling is defined by

$$q' = bq, \tag{A1}$$

and the field rescaling is then

$$\varphi'_{q'} = \zeta^{-1}\varphi_q, \tag{A2}$$

with the rescaling parameter ζ to be determined later. Note that in this appendix, for convenience, we have introduced the shorthand φ_q for $\varphi_{\boldsymbol{k}}$, and $\boldsymbol{n}$ instead of $\hat{\boldsymbol{n}}$ which we used in the main text.

The existence of the hypersphere in momentum space obeying $|\boldsymbol{k}| = k_0$ plays a profound role in the application of the RG to the Brazovskii model not seen in the usual application to the φ^4 theory of critical phenomena where we eliminate shells surrounding a single point (or a few points) in $\boldsymbol{k}$ space.

To see this, let us consider the quartic interaction. After the mode elimination at zero loops (or tree level), it is given by

$$\mathbf{X} \equiv \frac{1}{4!}\int_{\boldsymbol{k}_1}^{<} \cdots \int_{\boldsymbol{k}_3}^{<} \varphi_{\boldsymbol{k}_1} \cdots \varphi_{\boldsymbol{k}_4} u(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_4)\theta(\tilde{\Lambda}_0/b - |q_4|), \tag{A3}$$

where $\int_{\boldsymbol{k}}^{<} \equiv \int_{|k-k_0|<\tilde{\Lambda}_0/b} d^d k/(2\pi)^d$, $\boldsymbol{k}_4 = -(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$, and $q_4 = k_4 - k_0$. The θ function in (A3) is caused by the condition that each momentum $\boldsymbol{k}_i$ has to lie within the shell of thickness $2\tilde{\Lambda}_0/b$ around the hypersphere. We now see the problem in implementing the second step in the RG transformation, momenta rescaling. Namely, the θ function after the RG rescaling is not the same function of the new (rescaled) variables as the θ function before the RG transformation was of the old variables;

$$\theta(\tilde{\Lambda}_0/b - |q_4|) = \theta(\tilde{\Lambda}_0 - b|q_4|) = \theta\left(\tilde{\Lambda}_0 - \left|\,|\sum_{i=1}^{3}(bk_0 + q_i')\boldsymbol{n}_i| + bk_0\right|\right)$$
$$\neq \theta(\tilde{\Lambda}_0 - |q_4'|)\,. \tag{A4}$$

where $\boldsymbol{n}_i = \boldsymbol{k}_i/|\boldsymbol{k}_i|$. Thus the interaction measure does not come back to its old form after the RG transformation due to the change $k_0 \to bk_0$.

Solution of this impasse is to use a soft cutoff for q_4,[9]

$$\theta(\tilde{\Lambda}_0 - |q_4|) \to e^{-|q_4|/\tilde{\Lambda}_0}\,.$$

With this replacement, and keeping k_0 terms and neglecting $O(q)$ terms, the tree level interaction (A3) becomes

$$\begin{aligned}\mathbf{X} &= \frac{1}{4!}\int_{\boldsymbol{k}_1}^{<}\cdots\int_{\boldsymbol{k}_3}^{<}\varphi_{\boldsymbol{k}_1}\cdots\varphi_{\boldsymbol{k}_4}u(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4)\exp\left(-\frac{b|q_4|}{\tilde{\Lambda}_0}\right)\\ &= \frac{1}{4!}\int_{\boldsymbol{k}_1}^{<}\cdots\int_{\boldsymbol{k}_3}^{<}\varphi_{\boldsymbol{k}_1}\cdots\varphi_{\boldsymbol{k}_4}u(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4)\exp\left(-\frac{bk_0}{\tilde{\Lambda}_0}||\boldsymbol{n}_1+\boldsymbol{n}_2+\boldsymbol{n}_3|-1|\right)\\ &= \frac{1}{4!}\int_{\boldsymbol{k}_1'}^{<}\cdots\int_{\boldsymbol{k}_3'}^{<}\varphi'_{\boldsymbol{k}_1'}\cdots\varphi'_{\boldsymbol{k}_4'}u'(\boldsymbol{k}_1',\ldots,\boldsymbol{k}_4')\exp\left(-\frac{k_0}{\tilde{\Lambda}_0}||\boldsymbol{n}_1+\boldsymbol{n}_2+\boldsymbol{n}_3|-1|\right),\end{aligned} \tag{A5}$$

where

$$u'(\boldsymbol{k}_1',\ldots,\boldsymbol{k}_4') = \zeta^4 b^{-3}\exp\left(-\frac{(b-1)k_0}{\tilde{\Lambda}_0}||\boldsymbol{n}_1+\boldsymbol{n}_2+\boldsymbol{n}_3|-1|\right)u(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4). \tag{A6}$$

Thus, by iteration of RG (in other words, for large b, that is at the fixed point), only coupling with momenta satisfying the condition

$$|\boldsymbol{n}_1+\boldsymbol{n}_2+\boldsymbol{n}_3| = 1 \tag{A7}$$

will remain finite.

In two dimensions, Eq. (A7) has three solutions:

1. $\boldsymbol{n}_3 = -\boldsymbol{n}_1$ (hence $\boldsymbol{n}_4 = -\boldsymbol{n}_2$),
2. $\boldsymbol{n}_3 = -\boldsymbol{n}_2$ (hence $\boldsymbol{n}_4 = -\boldsymbol{n}_1$),
3. $\boldsymbol{n}_2 = -\boldsymbol{n}_1$ (hence $\boldsymbol{n}_4 = -\boldsymbol{n}_3$).

Thus in $d = 2$, the four vectors must be equal and opposite in pairs [as in Fig. 5(a)] when their magnitude goes to k_0.

In $d = 3$, Eq. (A7) has an additional continuum of solutions. Suppose $\boldsymbol{n}_1 + \boldsymbol{n}_3 \neq 0$. Then the sum of the vectors $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ lies on the plane they define and

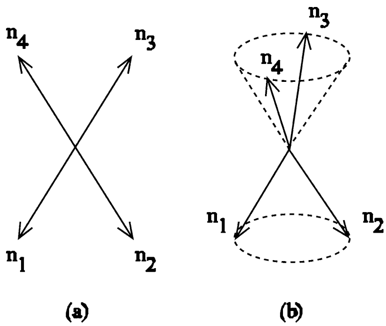

Fig. 5. Allowed momenta at the fixed point of RG transformation in $d = 2$ (a), and in $d = 3$ with $\boldsymbol{n}_1 + \boldsymbol{n}_2 \neq 0(b)$.

bisects the angle between them. The vectors $\boldsymbol{n}_3$ and $\boldsymbol{n}_4$ can lie anywhere on the cone generated by rotating $\boldsymbol{n}_3$ and $\boldsymbol{n}_4$ around their sum [see Fig. 5(b)]. Then

$$\boldsymbol{n}_1 + \boldsymbol{n}_2 + \boldsymbol{n}_3 + \boldsymbol{n}_4 = 0$$

as required by the momentum conservation, and the condition (A7) is satisfied at the same time. In the particular case where $\boldsymbol{n}_1 + \boldsymbol{n}_3 = 0, \boldsymbol{n}_2$ and $\boldsymbol{n}_4$ must be mutually opposite as in $d = 2$.

The RG used in Ref. 7 considers only the exceptional case when $\boldsymbol{n}_1 + \boldsymbol{n}_3 = 0$. Being stated in another way, the RG calculations of Ref. 7 assume that dependence on the angle between the planes containing the pair $(\boldsymbol{n}_1, \boldsymbol{n}_2)$ and $(\boldsymbol{n}_3, \boldsymbol{n}_4)$ can be ignored. The coupling u then depends only on the variable $z = \boldsymbol{n}_1 \cdot \boldsymbol{n}_2$, becoming the full equivalent of the Landau parameter F in Landau's Fermi-liquid theory.[9] We follow this strategy in the main text of the present paper.

We will choose the scaling parameter ζ so that the coefficient $\bar{\xi}_0$ of the free part of the action [Eq. (2) with (3) in the main text] is kept constant. Then we find

$$\zeta = b^{3/2}, \tag{A8}$$

that is, the scaling dimension of the field $\varphi(\boldsymbol{r})$ in real space is $d_\varphi = d - 3/2$. The relation (A6) with (A8) demonstrate that the coupling constant $u(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_4)$ is a relevant coupling. Repeating the preceding analysis, we can also draw the similar conclusions for the higher order coupling constants.

Finally, we remark that a free action of the form (2) with either[7]

$$G_0^{-1}(\boldsymbol{k}) = r_0 + \tilde{\xi}_0^4 \left(k^2 - k_0^2\right)^2$$

or[25]

$$G_0^{-1}(\boldsymbol{k}) = r_0 + \frac{a}{k^2} + ck^2$$

does not reproduce itself after the RG rescaling transformation. This is why we have started with the free part of the form (3) in our RG formulation.

ACKNOWLEDGMENTS

The author is grateful to F. Corberi, H. Diehl, T. Imaeda, S. Puri and G. Tarjus for very informative conversations on various topics considered in this paper.

REFERENCES

1. S. A. Brazovskii, Phase transition of an isotropic system to a nonuniform state. *Zh. Eksp. Theor. Fiz.* **68**(1):175–185 (1975) [*Sov. Phys. JETP* **41**(1):85–89 (1975)].
2. S. A. Brazovskii and S. G. Dmitriev, Phase transitions in cholesteric liquid crystals. *Zh. Eksp. Theor. Fiz.* **69**(9):979–989 (1975) [*Sov. Phys. JETP* **42**(3):497–502 (1976)].
3. J. Swift, Fluctuations near the nematic-smectic-C phase transition. *Phys. Rev. A* **14**(6):2274–2277 (1976).
4. R. F. Sawyer, Condensed π^- phase in neutron-star matter. *Phys. Rev. Lett.* **29**(6):382–385 (1972).
5. J. Swift and P. C. Hohenberg, Hydrodynamic fluctuations at the convective instability. *Phys. Rev. A* **15**(1):319–328 (1977).
6. L. Leibler, Theory of microphase separation in block copolymers. *Macromolecules* **13**(6):1602–1617 (1980).
7. P. C. Hohenberg and J. B. Swift, Metastability in fluctuation-driven first-order transitions: Nucleation of lamellar phases. *Phys. Rev. E* **52**(2):1828–1845 (1995).
8. K. G. Wilson and J. B. Kogut, The renormalization group and the ϵ expansion. *Phys. Rep.* **12C**(2):7–199 (1974); S. K. Ma, *Modern Theory of Critical Phenomena* (Benjamin-Cummings, Reading, MA, 1976).
9. R. Shankar, Renormalization-group approach to interacting fermions. *Rev. Mod. Phys.* **66**(1):129–192 (1994).
10. F. J. Wegner and A. Houghton, Renormalization group equation for critical phenomena. *Phys. Rev. A* **8**(1):401–412 (1973).
11. For a review of recent development, see C. Bagnuls and C. Bervillier, Exact renormalization group equations. An introductory review. *Phys. Rep.* **348**(1–2):91–157 (2001); J. Berges, N. Tetradis, and C. Wetterich, Non-perturbative renormalization flow in quantum field theory and statistical physics. *ibid.* **363**(4–6):223–386 (2002).
12. P. Kopietz, Two-loop β-function from the exact renormalization group. *Nucl. Phys. B* **595** (1–2):493–518 (2001); P. Kopietz and T. Busche, Exact renormalization group flow equations for non-relativistic fermions: Scaling towards the Fermi surface. *Phys. Rev. B* **64**(15):155101–1–14 (2001).
13. S. Ledowski and P. Kopietz, An exact integral equation for the renormalized Fermi surface. *J. Phys.: Condens. Matter* **15**(27):4779–4787 (2003).
14. T. R. Morris, The exact renormalization group and approximate solutions. *Int. J. Mod. Phys. A* **9**(14):2411–2449 (1994).
15. T. R. Morris, Momentum scale expansion of sharp cutoff flow equations. *Nucl. Phys. B* **458**(3):477–503 (1996).
16. K. Kawasaki, T. Imaeda, and J. D. Gunton, Coarse-grained Helmholtz free energy functional, in *Perspectives in Statistical Physics*, H. J. Raveché (ed.) (North-Holland Pub. Co., Amsterdam, 1981), Chap. 12.

17. K. Binder, in *Phase Transformations in Materials*, P. Haasen (ed.) (VCH, Weinheim, 1991), Chap. 7.
18. See, for example, N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group* (Addison-Wesley, Reading, MA, 1992), Chap. 12.
19. A. Díaz-Guilera, Dynamic renormalization group approach to self-organized critical phenomena. *Europhys. Lett.* **26**(3):177–182 (1994).
20. (a) K. R. Elder, J. Vinñals, and M. Grant, Ordering dynamics in two-dimensional stochastic Swift-Hohenberg equation. *Phys. Rev. Lett.* **68**(20):3024–3027 (1992); Dynamic scaling and quasiordered states in the two-dimensional Swift-Hohenberg equation. *Phys. Rev. A*. **46**(12):7618–7629 (1992); (b) Y. Shiwa, H. Matsunaga, M. Yoshikawa, and H. Yoshitomi, Fluctuation-induced first-order transition and dynamic scaling in Rayleigh-Bénard convection. *Phys. Rev. E* **49**(3):2082–2086 (1994); Y. Shiwa, T. Taneike, and Y. Yokojima, Scaling behavior of block copolymers in spontaneous growth of lamellar domains. *Phys. Rev. Lett.* **77**(21):4378–4381 (1996); (c) Q. Hou, S. Sasa, and N. Goldenfeld, Dynamic scaling behavior of the Swift-Hohenberg equation following a quench to the modulated state. *Physica A* **239**(1–3):219–226 (1997); (d) J. J. Christensen and A. J. Bray, Pattern dynamics of Rayleigh-Bénard convective rolls and weakly segregated di-block copolymers. *Phys. Rev. E* **58**(5):5364–5370 (1998); (e) D. Boyer and J. Viñals, Domain coarsening of stripe patterns close to onset. *ibid.* **64**(5):050101(R)-1–4 (2001).
21. M. E. Fisher and A. N. Berker, Scaling for first-order phase transitions in thermodynamic and finite systems. *Phys. Rev. B* **26**(5):2507–2513 (1982); A. J. Bray, Renormalization-group approach to domain-growth scaling. *ibid.* **41**(10):6724–6732 (1990).
22. Reference [20b] given above; see also K. R. Elder and M. Grant, Singular perturbation theory for phase-front dynamics and pattern selection. *J. Phys. A* **23**(16):L803–L808 (1990).
23. U. Marini Bettolo Marconi and F. Corberi, Time-dependent Ginzburg-Landau equation for an N-component model of self-assembled fluids. *Europhys. Lett.* **30**(6):349–354 (1995).
24. C. Harrison, Z. Cheng, S. Sethuraman, D. A. Huse, P. M. Chaikin, D. A. Vega, J. M. Sebastian, R. A. Register, and D. H. Adamson, Dynamics of pattern coarsening in a two-dimensional smectic system. *Phys. Rev. E* **66**(1):011706–1–27 (2002), and references cited therein; see also H. Qian and G. F. Mazenko, Defect structures in the growth kinetics of the Swift-Hohenberg model. *ibid.* **67**(3):036102-1–12 (2003).
25. T. Ohta and K. Kawasaki, Equilibrium morphology of block copolymer melts. *Macromolecules* **19**(10):2621–2632 (1986).